Appendix: Additional Support for the Assumption $\beta > \gamma$

Here, we report some additional analysis that provide further support for the assumption $\beta > \gamma$, which is required for the existence of a pure-strategy Markov perfect equilibrium in our paper. Recall that β is the quality ratio of the low-quality product to the high-quality product, and γ is the customer discount factor, the assumption $\beta > \gamma$ implies that customers value more on purchasing the low-quality product immediately than waiting to purchase the high-quality product in the future when the two products are equally priced at the net present value. We show that if the firms can choose their quality levels, then the low-quality firm will never choose a quality level β smaller than γ, when the two firms subsequently engage in a dynamic pricing competition with two periods.

In order to consider quality competition between the two firms, we expand the game to include a period 0 where the firms compete on their quality levels. After the firms select their quality levels, they engage in a dynamic pricing competition as defined in the main body of our paper. For simplicity, we assume that each firm changes price only once (i.e., $T = 2$). The main difficulty of analyzing the quality competition game, in addition to the dynamic pricing competition, is the lack of a pure-strategy Nash equilibrium in the dynamic pricing competition game when $\beta \le \gamma$. Therefore, it is not immediately clear how to apply a standard backward induction approach to analyze the game. In this note, we adopt an alternative approach as follows. We consider an arbitrary equilibrium strategy in the three periods (quality competition in period 0, and price competition in periods 1 and 2). We show that, if $\beta < \gamma$, the lower-quality firm can always increase its quality level by a small amount and improve its profit.

Let (β, p_1, p_2) be an equilibrium strategy in the three periods, where $p_1 = (p_{1,H}, p_{1,L})$ and $p_2 = (p_{2,H}, p_{2,L})$ are the price pairs in periods 1 and 2, respectively. A customer with valuation θ purchases product L in the first period if

$$\beta\theta - p_{1,L} \ge \max\{\theta - p_{1,H}, \gamma(\theta - p_{2,H}), \gamma(\beta\theta - p_{2,L}), 0\}.$$

Hence, the demand for product L in period 1 for given (β, p_1, p_2), $d_{1,L}(\beta, p_1, p_2)$, is determined by

$$d_{1,L}(\beta, p_1, p_2) = \left[\min\left\{\frac{p_{1,H} - p_{1,L}}{1-\beta}, \frac{\gamma p_{2,H} - p_{1,L}}{\gamma - \beta}\right\} - \frac{p_{1,L} - \gamma p_{2,L}}{\beta(1-\gamma)}\right]^+.$$

Similarly, let $d_{2,L}(\beta, p_1, p_2)$ denote the demand for product L in period 2 given (β, p_1, p_2). Then

$$d_{2,L}(\beta, p_1, p_2) = \left[\min\left\{\frac{p_{1,H} - \gamma p_{2,L}}{1-\gamma\beta}, \frac{p_{2,H} - p_{2,L}}{1-\beta}, \frac{p_{1,L} - \gamma p_{2,L}}{\beta(1-\gamma)}\right\} - \frac{p_{2,L}}{\beta}\right]^+.$$

Let $\pi_L(\beta, p_1, p_2)$ be firm L's profit for given (β, p_1, p_2), then

$$\pi_L(\beta, p_1, p_2) = (p_{1,L} - \beta c) d_{1,L}(\beta, p_1, p_2) + \alpha(p_{2,L} - \beta c) d_{2,L}(\beta, p_1, p_2).$$

We now consider a strategy $(\hat\beta, \hat p_1, \hat p_2)$ where $\beta < \hat\beta = \beta + \epsilon < \gamma$, and $\hat p_1 = \left(p_{1,H}, \frac{\hat\beta p_{1,L}}{\beta}\right)$, $\hat p_2 = \left(p_{2,H}, \frac{\hat\beta p_{2,L}}{\beta}\right)$. Since $p_{i,H} > p_{i,L}$, there always exists $\epsilon > 0$ such that $p_{i,H} - \frac{\hat\beta}{\beta} p_{i,L} > 0$, $i = 1, 2$. Under this new strategy, firm L's demand in period 1 and period 2 are, respectively

$$d_{1,L}(\hat\beta, \hat p_1, \hat p_2) = \left[\min\left\{\frac{p_{1,H} - \hat\beta p_{1,L}/\beta}{1-\hat\beta}, \frac{\gamma p_{2,H} - \hat\beta p_{1,L}/\beta}{\gamma - \hat\beta}\right\} - \frac{p_{1,L} - \gamma p_{2,L}}{\beta(1-\gamma)}\right]^+,$$

$$d_{2,L}(\hat\beta, \hat p_1, \hat p_2) = \left[\min\left\{\frac{p_{1,H} - \gamma\hat\beta p_{2,L}/\beta}{1-\gamma\hat\beta}, \frac{p_{2,H} - \hat\beta p_{2,L}/\beta}{1-\hat\beta}, \frac{p_{1,L} - \gamma p_{2,L}}{\beta(1-\gamma)}\right\} - \frac{p_{2,L}}{\beta}\right]^+.$$

The associated profit is given by

$$\pi_L(\hat{\beta}, \hat{p}_1, \hat{p}_2) = \left(\frac{\hat{\beta} p_{1,L}}{\beta} - \beta c\right) d_{1,L}(\hat{\beta}, \hat{p}_1, \hat{p}_2) + \alpha \left(\frac{\hat{\beta} p_{2,L}}{\beta} - \beta c\right) d_{2,L}(\hat{\beta}, \hat{p}_1, \hat{p}_2).$$

Note that $\gamma p_{2,H} - p_{1,L} > 0$; otherwise $\beta\theta - p_{1,L} < \gamma(\theta - p_{2,H})$ for any θ since $\gamma > \beta$, implying no demand for firm L in period 1. However, this cannot happen for an equilibrium price strategy (p_1, p_2). Similarly, we can argue that

$$\begin{aligned}
\frac{p_{1,H} - p_{1,L}}{1-\beta} &< \frac{p_{1,H} - \hat{\beta} p_{1,L}/\beta}{1-\hat{\beta}}, \\
\frac{p_{2,H} - p_{2,L}}{1-\beta} &< \frac{p_{2,H} - \hat{\beta} p_{2,L}/\beta}{1-\hat{\beta}}, \\
\frac{p_{1,H} - \gamma p_{2,L}}{1-\gamma\beta} &< \frac{p_{1,H} - \gamma\hat{\beta} p_{2,L}/\beta}{1-\gamma\hat{\beta}}, \\
\frac{\gamma p_{2,H} - p_{1,L}}{\gamma-\beta} &< \frac{\gamma p_{2,H} - \hat{\beta} p_{1,L}/\beta}{\gamma-\hat{\beta}}.
\end{aligned}$$

Therefore, we have $d_{i,L}(\beta, p_1, p_2) < d_{i,L}(\hat{\beta}, \hat{p}_1, \hat{p}_2)$, $i = 1, 2$. It then immediately follows with

$$\pi_L(\beta, p_1, p_2) < \pi_L\left(\hat{\beta}, \hat{p}_1, \hat{p}_2\right). \tag{1}$$

This is sufficient to show that a quality level of $\beta < \gamma$ cannot be sustained as a Nash equilibrium if firm L could choose the quality level.